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Preface

The Diagrams Graduate Symposium (GS) provides Master and Doctoral students with an opportunity to present their work and get feedback from established researchers in the field. It is also a supportive environment for students to network and make contact with potential future colleagues or employers. The GS was an integral part of the Diagrams 2018 programme. As in previous years, lively discussions led to suggestions about the students' on-going research, and allowed experienced participants to hear fresh ideas and view some of the new trends in the field.

This year's GS proceedings constitute eleven student papers. These are short papers that describe the students' research. Each paper was reviewed by two distinguished scholars whom provided constructive and detailed feedback to the students. Further to the reviewer's feedback, students were assigned mentors for the duration of the conference. An important part of the mentor's role was to provide constructive feedback and advice in response to their student's presentation. Students, therefore, received valuable and construction feedback for both their paper and presentation.

The success of this year's GS arose for a number of important reasons. The diversity of the students assured, in part, the quality of their research. For example, students represented nine different universities from five different countries across three continents. Diagrams 2018 also received valuable funding from the National Science Foundation (NSF) facilitating graduates from the USA to make the long journey to Edinburgh. The conference organising committee worked hard to ensure the smooth running of each day's events. Finally, and most important of all, the students themselves ensured a vibrant, lively and engaging GS for all who attended.

June 2018, Andrew Blake

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Table of Contents

	<i>Pages</i>
Investigating Diagrammatic Reasoning with Artificial and Biological Neural Networks Duo Wang, University of Cambridge, UK	1 to 3
'Diagrams': A Hybrid Visual Information Representation and Reasoning Paradigm Towards Video Analysis Chayanika Deka Nath, Tezpur University, India	4 to 6
Diagrammatic Metro Maps: Navigational Problems and their Detection and Remediation Peter Lloyd, University of Kent, UK	7 to 9
Diagrammatic Definitions of Causal Claims Dean Mchugh, University of Amsterdam, Netherlands	10 to 12
Automated Visualization of Grouped Networks Almas Baimagambetov, University of Brighton, UK	13 to 15
Using Diagrammatic Proofs to Investigate High-Level Mathematical Thinking Josephine Relaford-Doyle, UC San Diego Cognitive Science, USA	16 to 18
Overcoming Graphical Fixedness: Scaffolding Comprehension for Unconventional Graphs Amy Fox, UC San Diego Cognitive Science, USA	19 to 21
The Effects of Prompts to Draw Diagrams in Engineering Sally Wu, University of Wisconsin-Madison, USA	22 to 24
The Relative Efficacy of Diagrams, Textual and Symbolic Logics on User Interpretation of Axioms Eisa Alharbi, University of Brighton, UK	25 to 27
Analysing Properties of Representations for Automated Heterogeneous Reasoning Aaron Stockdill, University of Cambridge, UK	28 to 30
Innovative Use of Diagrams and Inference in Wallis Erika Rita Ortiz, National University of Cordoba, Argentina	31 to 33

Investigating diagrammatic reasoning with artificial and biological neural networks

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1 Introduction

Diagrams in mechanised reasoning systems are typically encoded into symbolic representations that can be easily processed with rule-based expert systems. This relies on human experts to define the framework of diagram-to-symbol mapping and the set of rules to reason with the symbols. We present a new method of using Deep artificial Neural Networks (DNN) to learn continuous, vector-form representations of diagrams without any human input, and entirely from datasets of diagrammatic reasoning problems. Based on this DNN, we developed a novel reasoning system, Euler-Net, to solve syllogisms with Euler diagrams. Euler-Net takes two or three Euler diagrams representing the premises in a syllogism as input, and outputs either a categorical (subset, intersection or disjoint) or diagrammatic conclusion (generating an Euler diagram representing the conclusion) to the syllogism. Euler-Net can achieve 99.5% accuracy for generating syllogism conclusion. We analyse the learned representations of the diagrams, and show that meaningful information can be extracted from such neural representations.

2 Completed work

We developed Euler-Net, a Deep Neural Network that solve syllogism reasoning tasks represented by 2-set or 3-set Euler diagrams. This work is described in more details in our accepted conference paper titled "Investigating diagrammatic reasoning with deep neural networks". Figure 1 shows an architecture overview of Euler-Net. Euler-Net consists of a Siamese convolutional network [1] that extracts high level feature representations from the input premise diagrams, and a reasoning network that make inference on extracted representations of the premise diagrams. The Euler-Net can either output the inference in the categorical form (e.g. $A \subset B$) or generate an Euler diagram that represent the inference result. Such Euler-Net achieves 99.5% accuracy on 2-set Euler diagram task and 99.4% on 3-set Euler diagram task.

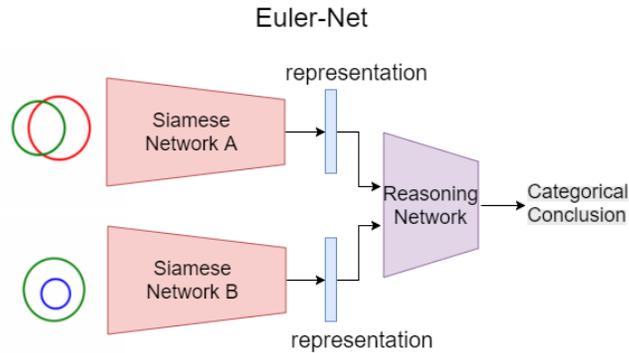


Fig. 1: Overview of the Euler-Net neural network architecture for Euler diagram syllogism reasoning.

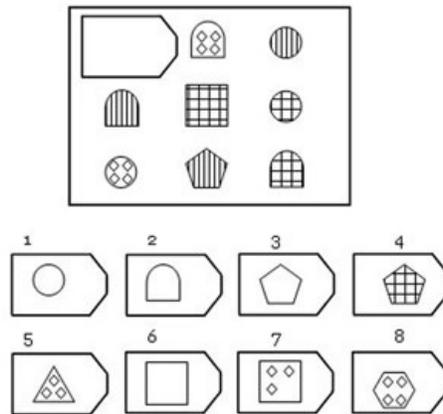


Fig. 2: An example of a diagrammatic reasoning question that is more difficult to formalise. In this diagram test question users' are asked to fill in the option that best fits the patterns in the diagram matrix.

3 Expected contribution

We propose that our framework can be applied to other types of diagrams, especially the ones we do not know how to formalise symbolically. We would like to firstly extend Euler-Net to be capable of reasoning with a full Euler or Spider diagram system. The extended Euler-Net should have the diagram proving capabilities of diagram theorem provers such as Edith [2] and Speedith [3]. We will then extend our DNN work to diagrams that are more difficult to formalize (e.g. well-used diagram with poorly defined semantics), such as diagram reasoning questions shown in figure 2.

Furthermore we propose to investigate the relation between our artificial DNN and human neural circuitry when performing diagrammatic reasoning. Research shows [4] that DNNs trained for image recognition tasks have high activity correlations with neural signals measured at human visual cortex when viewing the same visual input that DNNs are processing. This motivates us to investigate if DNNs trained for diagrammatic reasoning tasks, which are essentially a combination of image recognition and logical reasoning tasks, are also correlated to the biological neural circuitry in our brain. While lower layers in DNN can be mapped to visual cortex, we expect higher layers of DNN to be mapped to cortex regions involved in reasoning. For this investigation, we designed a functional Magnetic Resonance Imaging (fMRI) experiment that measures subject’s neural activities when they are reasoning with Euler diagrams.

Applying DNN on diagrammatic reasoning and investigating its correlation with biological neural circuitry can be potentially influential. DNN provides a way to encode these types of diagrams into neural codes that can be visualised, understood and even operated symbolically. Studying DNNs and their correlation with biological neural networks can allow us to discover inner patterns of diagrammatic reasoning in the human brain. For example, we can investigate if both DNN and humans learn similar heuristics that facilitate reasoning. Such discovery can provide insights for more effective design of diagrammatic reasoning systems.

4 Aspects needing advice

We would like to ask for advices for two aspects of the work. Firstly we would like to ask for advices on identifying types of diagrams that are difficult to formalize but yet have good values in term of research or application. This will help us prioritize our research on particular types of diagrams. Secondly, feedback is needed for choices of diagrams and reasoning problems in fMRI design, and also particular theoretical aspects of diagrammatic reasoning that are worth investigating in a neuroscience and mathematical modeling way.

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‘Diagrams’: A Hybrid Visual Information Representation and Reasoning Paradigm Towards Video Analysis

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1 Approach Undertaken

The goal is to forward a hybrid approach combining diagrammatic reasoning and qualitative reasoning towards a comprehensive representation for video analysis. Diagrams allow explicit relational representation of spatial entities and their perception through diagrammatic representation and reasoning (DR) techniques [9]. Qualitative spatial and temporal reasoning (QSTR) [8] is an established area boosting qualitative information abstractions over spatial substrate for everyday reasoning. Power of heterogeneous framework, combining diagrams and formal logic has influenced multidisciplinary research findings; mathematical theorem prover [14], spatial problem solver [4]. A combined diagrammatic and sentential representation is suggested by Gottfried in [10] to enrich QSTR for results within confined relational subset. Freska introduced need of comparison between formal and DR processes for same underlying problems. [11] established conceptual knowledge as a common language generalizing formal and diagrammatic approaches. Motivated by these established facts, the authors aim to unify conceptual and formal problem solving techniques for video data analysis. Figure 1 shows a conceptual architecture of the proposed approach

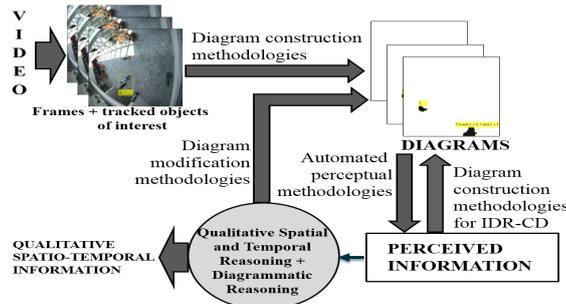


Fig. 1. Conceptual architecture of proposed methodology.

2 Work completed

‘Diagrams’: *The Hybrid Methodology* : *Diagrams* are 2-D image matrix representation (x-y axes coinciding camera axes) of video frames with diagrammatic objects representing objects of interest in the video frames along with certain automatically created objects representing spatio-temporal relations like distance and displacement directions {close polygons (tracked objects of interest), rays (objects’ direction of displacement) and lines (distance among object pairs)}. Objects’ properties and their relations are maintained. Minimum-maximum extends of polygons along axes and length of lines, specifies the associated *object properties*. *Object relations* include: Allen’s interval relations (IA) [1], relative positions (RP), relative distances (RDt) $\{-,+,0\}$, displacement directions (Di) and relative displacement directions (RD_i) derived from QD₈ [5] among polygon pairs. Fig 2 shows the IA relations, relative positions and QD₈ relations.

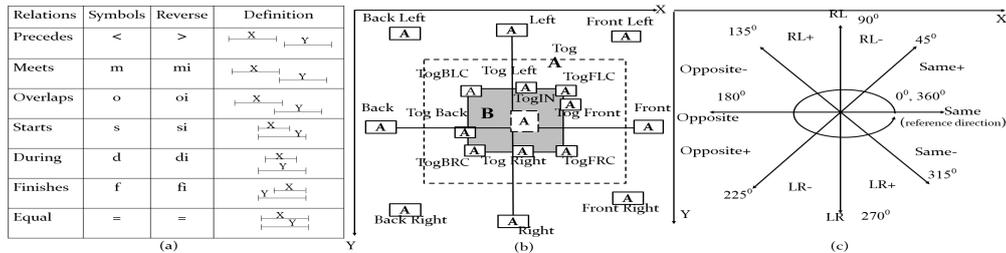


Fig. 2. (a) Allen’s interval relations, (b) 18 basic relative position relations, and (c) 8 basic QD₈ relations.

QSTR and DR techniques are implemented on diagrams through methodologies involving diagram creation, perceptual and diagram modification for spatio-temporal knowledge acquisition.

Diagram construction methodologies are defined to construct diagrams corresponding to video frames with tracked objects of interest. A sequence of diagrams called *key diagrams* (\mathcal{K}) are selected with difference in relative position and relative distance among interested object pairs. Pair of \mathcal{K} s are sequentially combined using IDR-OR operator [2] for *IDR-combined diagrams* (IDR-CD).

Automatic perceptual methodologies perceive visual information from diagrams and IDR-CDs. During *diagram creation* and *modification* perceptual information are manipulated for automation of object relations. Qualitative relations are perceived through analysis of quantitative object properties. *IA relations* among object pairs are the core for visualizing *relative positions* and initiating diagram modification for *distance* and *displacement relations*.

Diagram modification techniques are introduced with abilities to endow new information through automatic insertion of new diagrammatic objects like ‘lines’ and ‘rays’ based on IA relations among associated polygon objects.

Experiments and Results: For proof of concept, the proposed technique is evaluated on videos of J-HMDB dataset¹ for recognition of selected verbs {catch, throw, shoot ball, push and pull up}. For evaluation of proposed QSTR and DR mechanism, tracking is achieved through manual labelling of objects of interest; focus is only to validate the proposed methodology, attempts at improvement of tracking being outside the scope of this work. A ‘diagram’ sequence is automated, resulting automatic abstraction of spatio-temporal relations among object pairs (A,B), $\langle \text{Di}(A), \text{Di}(B), \text{RP}, \text{RD}_i, \text{RD}_t \rangle$. Motivated by event-activity modelling approach [3], a sequence of short term activities (STA) is formulated and considered as feature to a standard SVM for activity classification. For example, figure 3 shows (a) video frames and associated key diagrams with IA relations and distance information (‘lines’ in blue) and (b) IDR-combined diagrams with direction of displacement information (red ‘rays’ depict displacement of object 1 and blue ‘rays’ depict displacement of object 2) in a *shoot ball* video from J-HMDB dataset. In the example, the abstracted sequence of relations among object 1 and 2 from the IDR-CDs are as shown in table 1. Based on these relations a sequence of STAs obtained is: { AB:TogIN-Tog, ABapart:TogLeft} which constituting the minimal sequence for *shoot ball* activity among object 1 and 2. ‘AB:TogIN-Tog’ infers that objects A,B both are in motion and move from completely inside (TogIN) position to partial overlap (TogLeft) position; ‘ABapart:TogLeft’ infers that objects A,B both are in motion and move from completely partial overlap position (TogLeft) to disjoint (Left) position.

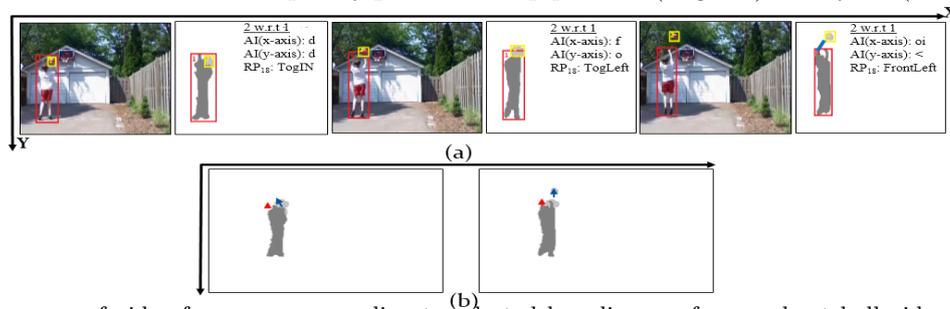


Fig. 3. Sequence of video frames corresponding to selected key diagram from a shoot ball video in JHMDB dataset, with (a) key diagrams with distance information and (b) IDR-CDs with displacement information.

Table 1. Perceived qualitative relations of object A w.r.t. object B in IDR-CDs of example in figure 3(b).

	Relations as a set of {Di of A, Di of B, RP of B w.r.t A in time frame ‘t’, RP of B w.r.t A in time frame ‘t+1’, RD _i of B w.r.t A , RD _t of B w.r.t A }
IDR-CD₁	{ RL , RL+ , TogIN, TogLeft, Same+, 0 }
IDR-CD₂	{ RL , RL , TogLeft, Left, Same, +ve }

Table 2 shows the performance of the proposed methodology with respect to ground truth provided by human observer; a rough comparison of recognition accuracy of selected activities with that of state-of-the-art performances is provided. Since, performance accuracy of the activities in published state-of-the-art techniques are computed over all the 21 verbs of J-HMDB dataset an accurate comparison of the recognition performance is not possible. However, as observed from the limited comparison with considered verbs from the J-HMDB dataset performance seems to be inspiring for the underlying

¹ <http://jhmdb.is.tue.mpg.de>.

methodology. Better performance may be achieved through consideration of more stronger feature vector or via other recognition techniques.

Table 2. Activity recognition performance on videos of J-HMDB dataset reported in terms of accuracy, precision, recall and F-score; per class accuracy comparison with state-of-the-art performances is presented.

Activities	STA sequence + SVM				Stacked feature vectors [12]	Pose-based CNN [7]	Dynamic programmed SVM [13]	GRP [6]	GRPw/o constraints [6]
	Accuracy	Precision	Recall	F-Score	Accuracy(APPROX.)				
Catch	87.2	70.0	70.0	70.0	38.0	51.0	22.0	50.0	56.0
Throw	95.3	100	80.0	88.9	35.0	38.0	18.0	72.0	72.0
Shoot ball	91.1	87.5	70.0	77.8	77.0	80.0	35.0	45.0	18.0
Push	97.6	90.9	100	95.2	81.0	95.0	60.0	75.0	75.0
Pull up	89.1	69.2	90.0	78.2	100	95.0	86.0	100	100

3 Expected Contributions

Human basically convey effective solutions based on mental maps or organization of problems. The proposed methodology is a step towards formalizing the use of diagrams in cognitive vision for representation and reasoning purpose. The authors strongly advocate general concept and perception about a spatio-temporal structure to be computationally effective over formal computation with detailed and complex organizational information. A novel approach of integrating DR and QSTR techniques is being presented for video data representation in a cognitive vision system. This hybrid strategy narrows the option of ambiguity in relational composition. The work presented shows how diagrams and ‘commonsense knowledge’ can be put together for a human like problem definition through: perception of information, endowing new information through diagram modification and inter diagrammatic reasoning. An application of the proposed methodology for activity recognition is being presented via evaluation of few videos from J-HMDB dataset and encouraging recognition results are obtained.

4 Aspects of the work on which advice is desired

A) Look forward to have **feedback** on the **representation** and **reasoning** framework. B) Seek **advice** on the **evaluation**. C) Look forward for a **discussion** on defining **formal language automata** to uplift the framework towards **precise activity recognition**.

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Diagrammatic Metro Maps: Navigational Problems and their Detection and Remediation

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Abstract. The diagrammatic map format is very widely used to represent the routes and services available to passengers of metro rail systems. Despite being extensively used for many decades, this cartographic format has only recently been subject to systematic theoretical and empirical research. There are still large unknowns in the field. This doctoral research project addresses one aspect. It seeks to characterise aspects of a metro map—other than the overall layout—that affect the usability of the map. In particular, it examines colour coding, and navigational hazards that may be produced by inadequately designed transfer points, junctions, and branches. In its second phase, the project will endeavour to utilise this empirical information in the development of software for the automated detection and remediation of such navigational problems in metro maps.

Keywords: Metro Maps, Colour Coding, Navigational Hazard, New York City Subway, Usability Testing, Automated Design.

1 Introduction

Topic. This research aims to characterise navigational problems in diagrammatic maps of metro rail networks, and to develop software tools to detect and, ultimately, fix those problems. The research is primarily in the field of computing but draws on other disciplines, namely psychology, graphic design, and information design for public transport.

Research gap (1): Usability. Metro maps are traditionally created in an artisanal manner without psychological theory or empirical studies of usability. Recently, the notion of evidence-based design of metro maps has emerged, combining psychological models of map cognition with empirical studies of map features and objective measures of usability. Systematic investigation of metro maps by scientific principles is comparatively recent (Guo, 2011). The main results yielded by contemporary usability studies (summarised by Roberts, 2014) are: that objective and subjective evaluations of usability are uncorrelated; that layout affects journey planning; that Beck’s octilinear layout is not a ‘gold standard’ for map layout in all cities, and that instead a framework of more general principles for map layout can be identified, involving the simplicity, coherence, and harmony of line trajectories, and the balance and topographicity of the map layout. Which aspects of a map have the most impact on objective usability is an open question

for empirical research. Factors likely to influence usability are: overall layout; colour coding; symbolism and layout of transfer stations; symbolism of non-transfer stations; junction layout; and positioning and typography of labels. What these investigations lack so far is a consideration of the ‘micro-design’ of localised features such as transfers and junctions, as opposed to the ‘macro-design’ of layout.

Research gap (2): Automated design. In parallel with research on map usability, the past fifteen years have seen progress on automated map design. Three main methods of automatically laying out metro maps have been proposed (see Wolff’s (2007) review). Stott & Rodgers (2004) employed a hill-climbing multi-criteria optimization, which is flexible and lets us experiment with different selections of criteria and weightings. Hong (2006) proposed a force-directed model, which is well established for pure diagrams but does not naturally lend itself to diagrams coupled to an underlying geography. Nöllenburg and Wolff (2006) used a mixed-integer programming (MIP) method, a general-purpose global optimisation procedure. A fourth can be noted: Anand et al.’s (2007) simulated annealing method. This work has so far been dominated by the problem of laying out the map, not localized design features. It has also lacked input from cognitive theories that have begun to emerge from the empirical work on usability.

Research Question. The research questions in my doctoral project are: 1) What are the major factors—besides layout—that affect usability in metro maps, and how can we characterise them with reference to psychological models of map reading? 2) How can hazards arising from those factors be algorithmically detected and rectified?

Approach so far. Guided by an outline psychological model of map reading, I made an empirical study of the effect of colour-coding and local navigational hazards in the NYC subway map. This used experimental variants of the official map, and 300 paid volunteers recruited through Mechanical Turk. The first analysis (Lloyd et al., 2018) reports on colour coding. Further phases in hand, on journey planning and navigational hazards, will yield information on features in maps that lead to misnavigation.

2 Work that has been completed

The first analysis established that, as predicted, the colour-coding of metro maps by individual route improves the accuracy with which a map user can trace a route through the map. Also, that local navigational hazards can have a major effect on the usability, and interact with the effect of



Figure 1 Trunk and route colour versions: where route-tracing ‘slips’

colour coding. Certain specific forms of transfer symbolism and junction layout have been shown to mislead users. Fig. 1 shows a simple case labelled ‘Slip F (Flip)’: in the trunk coloured map, more than in the route-coloured one, users are more likely to ‘slip’ between routes where the route lines ‘flip’. Full details are given by Lloyd et al. (2018). Where journey planning involves complex cognition, however, the effect is apparently reversed as the clutter of the route colours impacts the cognitive load.

3 Going forwards

Expected contributions of the research. From the first part of this research project, I hope to get new information about hazards in the design of diagrammatic metro maps, which will be of use to map designers and prove useful to developers of automated design software. From the second part, I hope eventually to have a software tool that would work as a ‘map checker’ (by analogy with spell checkers and grammar checkers), scanning a diagrammatic map and reporting on parts that are recognisably hazardous.

Current problem. A key problem is to find generalisable algorithms for recognising navigational hazards, such as misleading transfers. The first approach will involve *ad hoc* methods for each hazard in a vector graphic. General-purpose recognition tools such as neural nets work with raster images but do not lend themselves to vector images.

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Research Report for Graduate Symposium: Diagrammatic Definitions of Causal Claims

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January 2017

Abstract. We present a class of diagrams in which to reason about causation. These diagrams are based on a formal semantics called ‘system semantics’, in which states of systems are related according to temporal succession. Arguing from straightforward examples, we provide the truth conditions for causal claims that one may make about these diagrams.

The Thesis Topic. Diagrams offer a natural and highly expressive means of depicting causal relations. Flowcharts are the ubiquitous example, but even more concerted work to analyse causal relations specifically employs an abundance of visual aids. Lewis (1974, 564), for instance, diagrammatically depicts similarity orderings over worlds, while Spirtes, Glymour, and Scheines (2000) and Pearl (2009) represent Bayes nets as directed acyclic graphs (DAGs).

My current research follows the diagrammatic tradition by presenting a class of diagrams in which to reason about causation. The bulk of the work consists in presenting a variety of cases in which diagrams can represent causal relations. Regarding the underlying formal apparatus, we construct these diagrams from a semantics called ‘system semantics’. It turns out that system semantics furnishes a purely diagrammatic characterisation of various kinds of causal dependence. Moreover, one may give a diagrammatic definitions of two more refined notions of ‘sometimes’ and ‘partial’ causation. There is, however, more to do—during the conference I look forward to discussing avenues for future work.

The Approach being Taken: System Semantics. To begin by analogy, system semantics aims to do for causal claims what Kripke semantics has achieved in the philosophical discussion of possible and necessary truth. Indeed, we modify Kripke semantics for modal logic to create diagrams called ‘systems’ that specify precisely how parts of possible worlds change through time. A system \mathbb{S} is a pair $\langle St, R \rangle$ composed of a set St of states and a relation R of temporal succession between them. Each state is a valuation of atomic sentences in the language of propositional logic. Given states $s_1, s_2, s_3, \dots, s_n$, the intuitive reading of $Rs_1s_2s_3 \dots s_n$ is that, if s_1 is the current state, then s_2 may be the next state, after which comes s_3 , after which comes \dots , after which comes s_n .

To illustrate, suppose we have two atomic sentences representing a switch being up (S), and a light being illuminated (L). Figure 1 represents the interaction between the switch and light (with states depicted as circles, accompanied

by the sentences that are true at them, and the succession relation depicted by arrows). The diagram of Figure 1 shows, for instance, that when the switch is up and light is off ($S, \neg L$) the system changes into the state where the switch is up and the light is on (S, L). And the top-left loop demonstrates that if the switch is down and light is off ($\neg S, \neg L$), then they remain so in the next state.

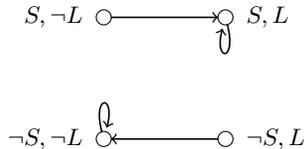


Fig. 1. System composed of a switch and light.

It turns out that one may give the truth-conditions for some causal claims purely in terms of these diagrams, as shown by work already completed.

Work that has Been Completed. The key work consists in giving the truth-conditions of causal claims in terms of system semantics. For instance, there are many causal claims one may make about the diagram of Figure 1. It seems the following causal claim should come out as true: “The switch being up is a necessary and sufficient cause of the light being on.” My current research proposes first adopting the following definition of minimal causation, before constructing more refined definitions from it.

Definition 1 (Minimal cause). *A is a minimal cause of B just in case*

- (1a) *some B-state leads to some $\neg B$ -state, or vice versa, some $\neg B$ -state leads to some B-state,*
- (1b) *some A-state leads to some B state, and*
- (1c) *some $\neg A$ -state leads to some $\neg B$ -state.*

We then add to the conditions above to define the following notions of necessary and sufficient causation.

Definition 2 (Necessary cause). *A is a necessary cause of B just in case*

- (2a) *A is a minimal cause of B, and*
- (2b) *every path¹ to every B-state comes from² some A-state.*

Definition 3 (Sufficient cause). *A is a sufficient cause of B just in case*

- (3a) *A is a minimal cause of B, and*
- (3b) *every path from every A-state leads to some B-state.*

The reader is invited to check that, according to definitions 2 and 3, S is indeed a necessary and sufficient cause of L in the system of Figure 1.

¹ By ‘path’ we mean the transitive closure of the succession relation.

² By the ‘coming from’ relation we mean the converse of the transitive closure of the succession relation.

The Expected Contributions of the Research. The main contribution of the research is to better understand the semantics of causal claims. In this endeavour, diagrams play an essential role. This is due to their ability to depict the models of system semantics, with which we may evaluate causal claims.

The research further allows one to consider new kinds of causal relations. For instance, one may introduce the following two notions of ‘sometimes’ and ‘partial’ causation by means of operations on diagrams.

Definition 4 (Sometimes relation). *A causal relation holds sometimes, in a system \mathbb{S} , just in case it holds by removing some (possibly no) arrows from \mathbb{S} .*

The purpose of introducing a notion of ‘sometimes’ causation is to capture causal reasoning in non-deterministic systems. This is a novel development as determinism is commonly assumed by many approaches to causal claims, such as Lewis (Menzies, 2017, §2.2) and the causal networks approach (Pearl, 2009; Spirtes et al., 2000). Similar to ‘sometimes’ causation, we define ‘partial’ causation like so.

Definition 5 (Partial relation). *A causal relation holds partially, in a system \mathbb{S} , just in case it holds by removing some (possibly no) states from \mathbb{S} .*

We call this kind of relation ‘partial’ as it need only hold in *part* of the model.

Aspects of the Work on which Advice is Desired. The diagrams considered by this research depict the modelling power of system semantics. Of course, we must invest some work just to provide a system-semantic representation of any given process, prior to analysing its causal relations. I would like advice on arguments for the capacity of system semantics to represent various kinds of processes we wish to model. For, given the widespread use of causal notions, such an argument is needed if system semantics is to fulfil its representational ambition.

The research has also not touched upon the metaphysical issues underlying system semantics; for instance, we took the notion of temporal succession to be unproblematic. A more comprehensive appraisal of system semantics must examine whether the choices of primitives made by system semantics fare better than those of other approaches to causality, such as the assumption of a similarity ordering over worlds made by Lewis (1974). A benefit of system semantics is that its metaphysical commitment—chiefly, an ontology of states related in time—is reasonably transparent, though to fully make the case for the philosophical adequacy of system semantics, one must still argue that those are sensible commitments to make.

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Automated Visualization of Grouped Networks

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Abstract. Euler diagrams are used for visualizing grouped data. They can be extended with graphs to visualize grouped network data. This project’s aim is to develop a tool capable of generating effective grouped network diagrams. This paper presents the foundation for such a tool in the form of a novel Euler diagram generation technique and discusses approaches to extend it for grouped network visualizations.

Keywords: information visualization, Euler diagrams, graph drawing

1 Introduction

The term ‘grouped network data’ refers to the data items in networks, where each item may also belong to groups. This type of data naturally arises in many areas, such as social networks, crime control and bioinformatics. Data items that are grouped into sets can be visualized by a variety of techniques with by far the most prominent being Euler diagrams [1]. Grouped network data can be visualized by a combination of an Euler diagram and a graph. As the amount of data increases, manual analysis and subsequent drawing of Euler diagrams and graphs quickly become more difficult. Automated layout tools can be of great benefit to data analysts when they wish to visualize grouped network data. However, visual tools for interrogating such data are seriously lacking, while existing attempts to automatically lay out Euler diagrams and graphs in combination yield suboptimal results.

A number of techniques that exist for drawing Euler diagrams all have problems. Hence, this project has two significant parts: a) develop a drawing technique to improve the effectiveness of Euler diagram layouts and b) extend the technique with graphs to produce effective grouped network diagrams. Both parts form original contributions of the project. Part a is completed and is outlined in section 2. The approach to complete part b is discussed in section 3.

2 iCurves: A Novel Euler Diagram Drawing Technique

An *Euler diagram* is a collection of closed curves, each of which has a label. Certain *well-formedness* properties have been identified and established to impact user understanding [3]. A diagram which does not break any well-formedness property is called *well-formed*. Rodgers et al. performed comparative user studies which revealed that the diagrams should be drawn well-formed if possible [3].

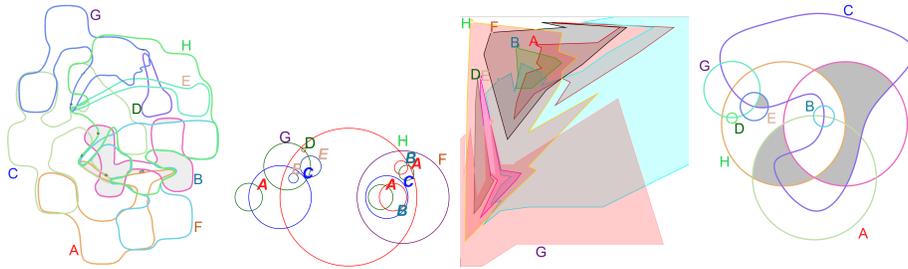


Fig. 1. Bubble Sets. **Fig. 2.** iCircles. **Fig. 3.** GED-L **Fig. 4.** iCurves.

Non-wellformed diagrams in Figures 1-3 visualize the same data and are drawn by existing state-of-the-art techniques (with available software) [1]. Our technique, iCurves, sets out to draw Euler diagrams that can represent any desired sets, whilst keeping the diagram well-formed, possibly with *extra* zones; these are shaded and represent the empty set, as in Venn diagrams. The well-formed diagram in Figure 4 was drawn by iCurves from the same data as in Figures 1-3.

Like existing methods, iCurves draws an Euler diagram from a list of set intersections. Such a list is split into a sequence of curve labels known as a *decomposition*, which determines the order in which curves are drawn. The use of a decomposition in Euler diagram generation is not new, but it can have a profound impact on the effectiveness of the diagram, as can the method used to draw the curves. So, in iCurves there are two major contributions of note: a) a new technique for drawing the curves, and b) a new strategy for producing a decomposition. The new technique for drawing curves allows iCurves to draw any description as a well-formed Euler diagram, whereas the decomposition strategy reduces the number of extra zones introduced in the final diagram.

3 Drawing Grouped Network Diagrams

Prior research has proposed layout techniques for grouped network diagrams by assigning *primary spatial rights* [2] to either the groups (Figure 5) or the network (Figure 6). It is posited that assigning such rights to groups may compromise the network and vice versa. As can be seen, the resulting diagrams are suboptimal: in Figure 5 there are unnecessary edge crossings, as well as edges that pass through non-incident nodes, in Figure 6 the Euler diagram has concurrent curves, the same set intersections occur more than once. The final diagram layout is intertwined with the layout of both the Euler diagram and the graph whose respective effective layouts are incompatible in most cases. Furthermore, in order to produce effective visualizations new combined layout properties need to be considered, such as the number of curves passed by an edge. In addition, it might be beneficial to preserve properties unique to Euler diagrams and graphs: well-formedness properties and straight line edges respectively. Hence, extending the iCurves technique in a way that the primary spatial rights [2] are not assigned

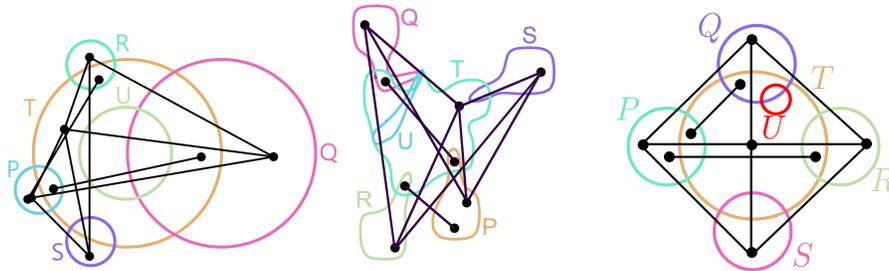


Fig. 5. iCurves (+graph).

Fig. 6. Bubble Sets.

Fig. 7. Hand-drawn.

to either the groups or the network represents a considerable challenge. Figure 7 depicts what form an effective visualization might take.

One possible approach to solve this problem is to define certain metrics based on the relationship between the groups and the network, which may include the number of items in a group, or the number of connections between items in two given groups. These metrics can inform the choices in the drawing algorithm to produce a potentially more effective visualization. Data sets from the SNAP (Stanford Network Analysis Project) collection can be analyzed for patterns to induce these metrics. Essentially, the technique is to draw diagrams that possess as many desirable properties as sensible within the identified constraints. The implementation of the extended technique will be evaluated against existing state-of-the-art methods. The results will be analyzed to inform the effectiveness of the new layout technique in comparison with existing techniques.

4 Conclusion and Future Work

Our novel technique, iCurves, generates well-formed diagrams, in contrast with existing techniques. Evaluation has shown that iCurves can outperform other techniques with respect to layout features known to be detrimental to task performance. We plan to extend iCurves to include graphs to draw grouped networks. It would be useful to gain advice on a) what aesthetics properties would be beneficial for diagrams to possess, b) what properties would constitute an effective diagram, and c) how best to evaluate the produced diagrams.

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Using Diagrammatic Proofs to Investigate High-Level Mathematical Thinking

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Abstract. This paper describes a novel method that uses diagrammatic proofs to investigate conceptualizations of infinite processes in adults. We use simple diagrammatic proofs to elicit high-level mathematical thinking from undergraduate students. By comparing the conclusions that math-trained and math-untrained participants draw from the image, we can better characterize informal notions of infinite processes and assess how they may differ from formal characterizations. Completed work has revealed important differences in how trained and untrained viewers use a diagrammatic proof about the infinite set of natural numbers. In ongoing work we are (1) applying the method to explore conceptualizations of infinite sums, and (2) using eye-tracking to explore how trained and untrained viewers inspect a diagrammatic proof in real time.

Keywords: Diagrammatic Proof, Mathematical Reasoning, Eye-Tracking

1 Thesis Topic and Approach

The machinery of formal mathematics allows trained mathematicians to know, with certainty, things that seem out of the reach of what humans can experience directly – for instance, that *all* members of an infinite set share a certain property, or what the “end result” of an infinite sum will be. In mathematics we arrive at these results through formal proofs, which use symbolic notation and complex mathematical transformations and are thus accessible only to the tiny portion of humans who are trained in formal mathematics. This raises the question – is such knowledge reserved only for mathematics experts? Or might there be ways of coming to this knowledge *outside* of formal mathematics?

In this project we use a novel method to examine how both trained and untrained adults reason about infinite processes. Rather than relying on formal proofs – which are accessible only to advanced mathematicians – we use diagrammatic proofs to elicit reasoning about infinite processes in both trained and untrained populations. Diagrammatic proofs exist for a variety of theorems in mathematics [1], and are often quite simple. For instance, Figure 1 shows a diagrammatic proof of a theorem involving an infinite set (left), and another involving an infinite sum (right). These images are relatively simple, and thus likely to be accessible to adults even without advanced mathematical training. By examining the conclusions that trained and untrained view-

ers draw from these images, we can better characterize informal notions of infinity and assess the extent to which knowledge about infinite processes may be attainable outside of formal mathematics.



Fig. 1. Left: Diagrammatic proof that for all natural numbers n , the sum of the first n odd numbers is equal to n^2 . Right: Diagrammatic proof that $\frac{1}{4} + (\frac{1}{4})^2 + (\frac{1}{4})^3 + \dots = \frac{1}{3}$.

2 Completed Work

We applied our method to investigate how adults reasoned about an infinite set [2]. We presented undergraduate participants with the diagrammatic proof in Figure 1 (left). This image was designed to demonstrate that, for all natural numbers n , the sum of the first n odd numbers is equal to n^2 , a statement which could be formally proven using mathematical induction. Half of our participants had received training in mathematical induction, and half were not familiar with the formal proof strategy. In a semi-structured interview, we explored the conclusions that participants drew from the image. We found that after working with the diagrammatic proof all participants, regardless of familiarity with formal mathematical induction, were willing to generalize the theorem to cases not depicted in the image like $n = 8$. However, only the participants who had been trained in formal mathematical induction were likely to express significant doubt that large-magnitude counterexamples to the theorem could exist. The majority of the untrained participants were willing to accept such a possibility, indicating that they hadn't generalized the theorem to *all* natural numbers. Thus, for untrained viewers the diagrammatic proof functioned similarly to a set of examples in that it allowed for inductive generalization, but did not provide the certainty associated with formal proof. These findings had implications for the epistemic status of diagrammatic proofs in mathematics, which we explore in [3].

3 Future Directions and Graduate Symposium Goals

Our method of using diagrammatic proofs to investigate high-level mathematical reasoning proved effective in revealing important differences between formal and informal notions of mathematical induction and the infinite set of natural numbers. More generally, this method is exciting in that it allows us to study high-level mathematical thinking *without* needing to restrict our population to trained mathematicians and mathematics students. We are interested in applying this method to investigate other mathematical concepts involving infinity, specifically infinite sums (like the one represented in Figure 1, right).

We are also interested in using eye-tracking to explore the unconscious processes that are involved in interpreting a diagrammatic proof in real-time, and how these differ between trained and untrained populations. Pilot data using the diagrammatic proof in Figure 1 (left) suggests interesting potential differences between how viewers who are and are not trained in formal mathematical induction inspect the image in real-time. In the first 60 seconds of inspecting the image, untrained viewers spent relatively more time fixating along the diagonal, with a symmetric distribution of fixations in the top-left and bottom-right triangles (Figure 2, left). Trained viewers fixated relatively less along the diagonal and typically spent much more time inspecting the top-left half of the image (Figure 2, right). This suggests that trained viewers may be more sensitive to the symmetry of the image, as well as the relational consistency between layers.

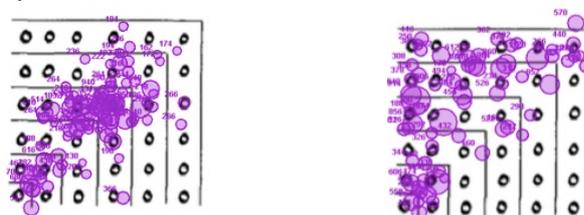


Fig. 2. Fixation patterns for a math-untrained viewer (left) and a math-trained viewer (right) in the first 60 seconds of inspecting the diagrammatic proof.

While my research is primarily concerned with mathematical reasoning, diagrammatic proofs are central in my work and I have much to learn from the Diagrams community. I am interested in receiving feedback on two aspects of my work. First, how can I best utilize eye-tracking as a complementary methodology? What are some of the key questions that eye-tracking could help me answer? Second, what are the current important questions or debates about diagrammatic proofs that could be informed by empirical study? I am particularly interested in the philosophical debate regarding the status of visual representations in mathematics, and would love to discuss how this debate could benefit from careful empirical work.

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Overcoming Graphical Fixedness: Scaffolding Comprehension for Unconventional Graphs

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Abstract. How do we make sense of a graph we’ve never seen before? Kosslyn [1] suggests we instantiate a hierarchically-organized graph schema. But what schema is triggered for novel representations? Pinker [2] speculates readers instantiate a “general graph schema” likely based on the coordinate system and “predominate graphical forms”. But what information does this schema contain, and how does it interact with prior knowledge of other graphical forms? Here we investigate the graph schema by exploring how learners read an unconventional representation. We ask: (1) What strategies do learners employ to make sense of an unconventional graph? (2) What explicit information might scaffold (self-directed) comprehension? (3) Can structuring graph-reading tasks as insight problems help overcome contradictory prior knowledge? (4) What is the time-course of mental model formation? (5) What inscriptions serve as the graphical framework, triggering a particular graph schema?

Keywords: Graph Comprehension, Statistical Graphs, Scaffolding Learning.

1 Background

As powerful as graphics may be in their communicative efficiency, they needn’t be *immediately* easy to understand [3]. This underlies much research in InfoVis: developing sophisticated representations for specialized tasks. The result is novel, unconventional representations computationally-suited to particular tasks, presenting interpretive challenges to untrained readers. In this project we use a simple but unconventional graph (Figure-1) The Triangular Model (TM) of Interval Relations [4] to explore how individuals make use of prior knowledge when reading new graph forms. We suspect the factors that drive ease of use in general, may *hinder* comprehension of unconventional representations. For the TM graph, our results suggest readers’ expectations for the structure of the coordinate system interfere with their ability to follow graphical cues provided by the graph’s diagonal gridlines: learners mischaracterize the system as Cartesian. We believe the substantial body of literature on insight problems (e.g. [5]) provides a promising direction for how we might support learners in overcoming this “graphical fixedness”.

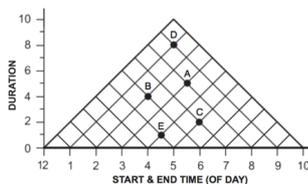


Figure 1: TM Graph

2 Summary of Studies and Methods

To address these questions, we've conducted three studies and planned two additional, utilizing qualitative and quantitative methods including: observation, interview, participatory design tasks, computer-based comprehension tasks, graph production tasks, eye-tracking, and participant narration of video-recorded mouse/gaze data.

Study One & Two: Observing Graph Reading & Evaluating Scaffolding. The results of our first two studies are reported in [6] to be presented at Diagrams '18. We began by asking the question: What strategies do students employ to make sense of an unconventional graph? We observed students solving scheduling problems with the Triangular Model (TM) graph, before challenging them to design instructional aids. We found most students mistakenly interpreted the graph as having a Cartesian coordinate system. [6]. In the follow-up interview, students produced both text and image instructions. In Study Two, we evaluated these student-suggested scaffolding techniques. Data from 316 STEM undergraduates revealed none of the scaffolds were sufficient to realize the computational efficiency of the TM [6]. Only an interactive image resulted in accuracy significantly better than the no-scaffold control. However, through subtle differences in materials we found students whose first question posed a 'mental impasse' (no correct answer if they misinterpreted the graph as Cartesian) had significantly improved performance. We suspect the novelty of the TM graph was insufficient in directing readers' attention to the salient differences between the graph and its more conventional alternative, resulting in a Cartesian misinterpretation. However, the mental impasse provided by questions in one set of materials directed readers' attention to their mistake. We address this hypothesis in Study Three.

Study Three: Evaluating Implicit vs. Explicit Scaffolding. In a factorial design we compared students' TM graph accuracy with a combination of explicit (none-control, static text/image, interactive image) and implicit (non-impasse-control, impasse) scaffolds. Data from 180 STEM undergraduates reveal that structuring problems to provide a mental impasse *did* significantly improve performance, though not more effectively than explicit scaffolds. We found substantial variance in accuracy, suggesting individual differences may play an important role in strategy. We are presently analyzing mouse-path recordings and inviting an additional group of students to provide talk-aloud narration of their mouse-path recordings. We are also gathering eye-tracking data for 60 students to explore the graph-inspection strategy by scaffold.

Study. Four: Timing the Mental Impasse. Next we explore the timing of mental model formation. In studies 1-3, we found students form an interpretation of the graph while solving the first problem, holding steady throughout the problem set despite implicit cues they are making errors. We hypothesize the presentation of an impasse

must be timed with this initial model formation. To test, we utilize the materials from Study 3 and assign STEM undergraduates to one of four timing conditions: (1) all-impasse, (2) first-impasse, (3) late-impasse and (4) no-impasse. We predict that to be effective, the impasse must be presented on the first question, while the students are forming a mental model of the graph system (Figure 2).

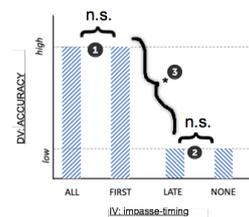


Fig 2. Study 4 Expected Results

Study Five: Defining the Graphical Framework. Finally we explore what features trigger a particular graph schema. We present readers with a TM graph featuring differing gridline/axis designs (Figure-3). We hypothesize the axes (not gridlines) trigger instantiation of a particular schema, therefore expecting significantly better performance by participants viewing diagonal (Figure 3 right) axis design.

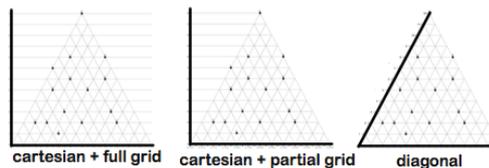


Fig 3. Alternative Axis/Grid designs

3 Doctoral Symposium Goals

As I will defend my proposal shortly after the workshop, I wish for this mentorship to help ground my plan in the broader context of current research in graph comprehension. I wish to connect with senior scholars to consider the relevance of this area of research in the landscape of research on external representations, and substantially strengthen the theoretical grounding of my specific research questions, shaping how I form the dissertation into a substantive basis for an ongoing research programme.

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The Effects of Prompts to Draw Diagrams in Engineering

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Abstract. Diagrams play a crucial role in advancing the engineering disciplines, yet engineering students rarely use diagrams as tools for learning. To address this issue, this study investigates the effects of implementing prompts to draw diagrams in an introductory undergraduate engineering course. Quantitative analyses showed that students' use of diagrams can affect learning outcomes on exams, and qualitative analyses revealed differences in how and why students used (or did not use) diagrams. Results provide insight into whether and how prompts to draw diagrams can help students learn concepts in the engineering classroom.

Keywords: learning strategy use, diagramming, engineering, post-secondary

1 Introduction

Drawing diagrams is a crucial practice for engineering professionals and instructors because it helps them explore and communicate concepts [1]. However, undergraduate students who intend to become engineers rarely draw diagrams to engage with concepts [2]. Students face difficulties in drawing diagrams [2, 3] and need instructional support to draw and reflect on diagrams, for example, via *drawing prompts* [4]. Such prompts simply ask or remind students to draw a diagram for specific problems or concepts. These prompts are easy to implement, especially in educational technologies.

The use of educational technologies has gained traction in engineering education because they engage students in solving problems while providing immediate feedback [5]. However, students can potentially identify correct answers in technologies, without engaging with the concepts [6]. To address this issue, drawing prompts on paper can enhance technologies by helping students engage with concepts more effectively [4, 7].

However, it is unknown whether such prompts are effective and how they engage students with diagrams in the context of an engineering course. Hence, I investigate the effectiveness of prompts in RQ1 (*Does prompting students to draw diagrams enhance learning outcomes in an undergraduate engineering course?*) and student use of prompts in RQ2 (*How do students engage with prompts and diagrams?*).

2 Method

To address these questions, I conducted a mixed-methods study that implemented prompts to draw diagrams while students solve problems in an educational technology.

I conducted the study in an introductory electrical engineering course on signal processing, with 189 students in Fall 2017 and 130 students in Spring 2018. The course was held in a technology-enhanced classroom with laptops, TV screens, mobile whiteboards, and circular tables that each seat six students. The course is designed as a “flipped” classroom in which students watch online video lectures prior to class. During the assigned class period, students solve problems presented by an educational technology. For each problem, students input a numerical answer and receive correctness feedback for up to three attempts. While students solve problems, the instructor, teaching assistant, and three undergraduate student coaches float around to answer questions.

I compare the Fall 2017 and Spring 2018 cohorts, to assess the effects of two instructional changes focused on diagrams in Spring 2018. First, in lecture videos, the instructor emphasized graphical understanding of concepts using diagrams, instead of formulas (a common instructional strategy). Moreover, the instructor advised students to draw diagrams to learn concepts. Second, students received diagram prompts embedded into in-class problems, as shown in Fig. 1. The prompts asked students to draw a diagram and then share it with others to discuss a strategy to solve the problem.

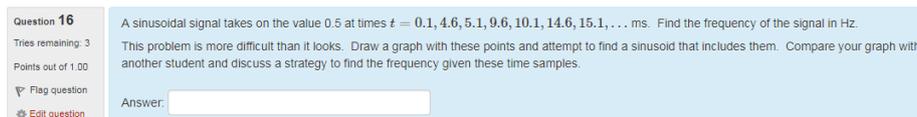


Fig. 1. Example in-class problem with a prompt to draw a diagram.

3 Preliminary Results

To address RQ1 (whether prompts enhance learning outcomes), I quantitatively assessed *performance data* as well as *survey data* on demographics and self-reported use and perceptions of diagrams. In the Fall 2017 cohort, an repeated-measures ANCOVA showed a significant interaction between exam performance and use of diagrams, $F(3, 507) = 3.39, p = .02, \eta^2 = .02$, and a marginal interaction of performance with perceived value of diagrams, $F(3, 507) = 2.25, p = .08$. Post-hoc analyses show that frequent use of diagrams increased performance on two exams, Exam 2 and Final Exam, and decreased performance on Exam 1 and 3. In the Spring 2018 cohort, grades on Exam 1 did not interact with the use ($p = .11$) or perceived value of diagrams ($p = .20$).

To address RQ2 (how students engage with diagrams), I assessed *survey data* that asked students how they used drawing prompts and *observational data* during in-class activities to assess how students engaged with problems, prompts, peers, and instructors. In the Fall 2017 cohort, 41 percent of students reported that they drew diagrams more than once a week. Most students (55%) found diagrams valuable, particularly for solving specific problems (e.g., “Phasors, discrete time frequency and convolution” or “Euclidean reasoning”). However, students may not use it as a general learning strategy (“I draw convolution diagrams because this is what you’re supposed to do”), while instructors were often observed drawing diagrams to solve problems. A majority of students (77%) used formulas and calculations to solve problems instead. In the Spring

2018 cohort, most students drew diagrams more than once a week (69%) and found them valuable (91%). Students drew diagrams to solve specific problems—but also to “better visualize the problem” and “visualize what is going on conceptually.” Or generally, in one student’s words: “I use this strategy to learn.” Further, almost all students used formulas more than once a week (94%) and found them valuable (97%).

4 Discussion

Findings from this study provide insight into how prompts to draw diagrams implemented in video lectures and in-class problems can help or hinder students’ performance in an engineering course. On a theoretical level, results reveal the potential role of diagrams in fostering engagement with concepts in a classroom. For instance, diagrams may enhance other strategies: drawing diagrams may have helped students in Spring 2018 use formulas more effectively. On a practical level, results yield recommendations for how to enhance technology-based activities in undergraduate engineering instruction, using a simple implementation of text-based prompts to draw diagrams.

Additional work is needed to generalize the results. This study is limited by its use of cohort comparison, though such designs are common in STEM education studies [5]. Further, final results are not reported; results from an ANCOVA that includes final grades and survey responses from the end of the Spring 2018 semester will be presented at the Diagrams Graduate Symposium. Findings will inform future studies that aim to promote drawing diagrams in the STEM classroom and other learning environments.

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The Relative Efficacy of Diagrams, Textual and Symbolic Logics on User Interpretation of Axioms

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Abstract. Concept diagrams were designed for developing ontologies. They visualise ontology specifications precisely through formal logic in a diagrammatic form. Ontologies are used by many stakeholders in different disciplines in order to share common knowledge. Ontology stakeholders use Web Ontology Language (OWL) and Description Logic (DL) for developing ontologies. The main aim of this research is to determine, via empirical study, whether diagrams can more effectively support novice users understanding than textual and symbolic logics. A further aim is to identify, via empirical study, how to choose between syntactically different diagrams when formulating logical axioms; this is a prerequisite to evaluating them compared to textual and symbolic notations.

Keywords: ontologies, axioms, diagrammatic styles, visualization

1 Introduction

An ontology can present a clear view of the structure of knowledge for a specific domain, and the relationships between information or data. With the abundance of data available in this information age, ontology engineering is becoming increasingly important. Ontologies can be used in any specific domain, therefore, many users are involved in ontology development. Ontology developers use formal textual or symbolic languages such as OWL or DL. Studies have shown that users face some challenges with OWL and DL. Concept diagrams is a formal visual language designed for developing ontologies. Concept diagrams can represent logical axioms in different diagrammatic styles. Having one style for expressing an axiom would help the development process. Comparing this diagrammatic style with textual and symbolic representations would establish whether there is an advantage of using this diagrammatic notation for developing ontologies. The first main contribution of this research, therefore, is to provide insight into choices between syntactically different, but semantically equivalent, concept diagrams. The second main contribution is to provide empirical evidence that concept diagrams are more effective than textual OWL and symbolic DL for representing a range of axiom types and deriving inferences from them. An axiom is a statement that is asserted to be true about a specific domain.

2 Empirical Studies

A series of empirical studies have been undertaken to observe the novice users' comprehension and interpretation of concept diagrams, OWL and DL. The results of these empirical studies will be a set of guidelines that will be linked to the aim of this research. In order to investigate this, participants were asked to answer questions about the information conveyed by notations. To determine whether there exist differences between the notations, two separate analysis processes were performed based on the accuracy rate and the time taken to answer each question. The participants were all novices, with no previous knowledge of concept diagrams, OWL or DL. Before each study, participants were given training, which was designed to introduce participants to the diagrams (or statements) and question types.

2.1 Comparing Equivalent Diagrammatic Styles

By appealing to ontology engineering, we identify commonly required semantic properties that require axiomatization. We systematically identify three different diagrammatic styles of axiomatizing these properties. The first does not use explicit quantification. The other two both use explicit quantification, but employ different diagrammatic devices to capture the required semantics. See figure 1 for an example of an *all values from* axiom type. If each element of C_1 is related under p , only to elements of C_2 (if they are related to anything), then C_1 is said to have *all values from* C_2 under p . We evaluated these competing styles by conducting a within-group empirical study, collecting performance data. The results showed that the *unquantified* style is significantly better than the other two styles. The results also indicated that using textual information such as negation and explicit quantification required more effort for interpreting the diagrams. A paper based on this chapter was published at the International Conference on the Theory and Application of Diagrams conference 2016 [1].

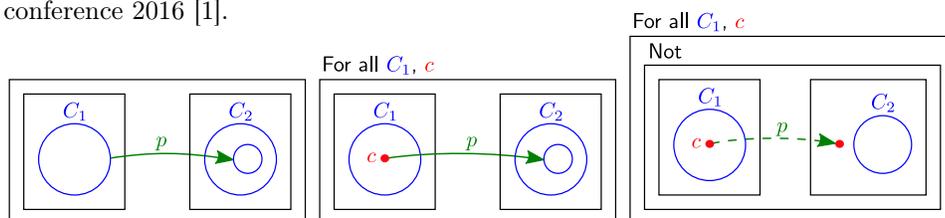
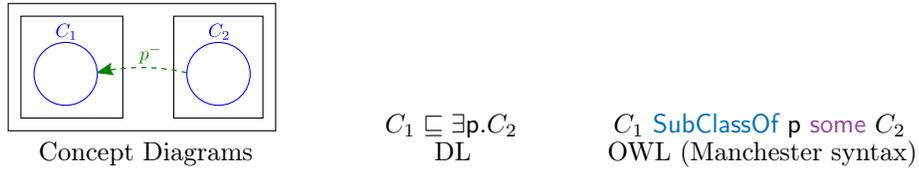


Fig. 1: Three diagrammatic styles representing *all values from* axiom type.

2.2 Understanding Common Axioms

Having determined that *unquantified* diagrammatic styles produced the best results, in this study we compared these diagrammatic styles with equivalent OWL and DL statements. The results suggest that concept diagrams best support user understanding of individual axioms, but we have also seen that OWL is better than DL in some cases. A paper based on this chapter was published at the Visual Languages and Human-Centric Computing 2017 conference [3]. Here is an example of representing the *some values from* axiom type in the three notations: $C_1 p$ some C_2 .



2.3 Understanding Axioms and their Entailments

A limit of the second study is its focus on understanding individual axioms. It is therefore important to build on these results to determine which notation – concept diagrams, OWL or DL – most effectively helped people understand the informational content of a collection of axioms and derive inferences from them. We focused on three tasks, *understanding* axioms, deriving *sound inferences* from them, and identifying *unsound inferences*. From ‘Pucks follow only Halfings’ and ‘All Halfings are Midgets’ in figure 2 we can deduce ‘Pucks follow only Midgets’ which is an example of making *sound inferences* from axioms. ‘Elves chase at least one Boggart’ is an example of making *unsound inferences* from axioms. The results again suggest that concept diagrams are more effective than OWL and DL. Surprisingly, we also found that OWL did not outperformed DL for these more complex tasks. A paper based partly on this study was published at the International Semantic Web Conference 2017 [2].

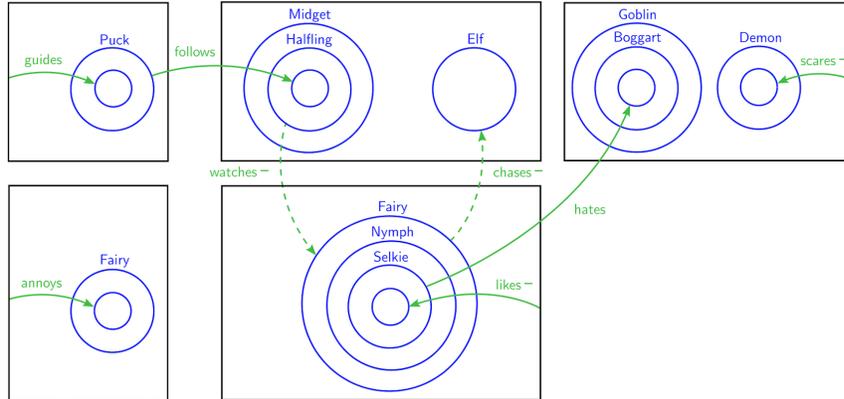


Fig. 2: A concept diagram expressing a collection of axioms.

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Analysing properties of representations for automated heterogeneous reasoning*

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Abstract. Human experts often use multiple representations when solving a problem to extract new information. While the benefits are increasingly clear, the mechanisms behind the choice of re-representation are poorly understood—even by those making the decision. We propose an analysis of problem representations from the perspective of *properties*, and discuss how varying the relative importance of these properties naturally guides the selection of alternative representations for a problem.

Introduction “Draw a figure” is common advice in problem solving, most famously given by Pólya on mathematics problems [3]. Particular representations may be able to present information in a “better” way by emphasising specific parts of the problem; humans often exploit this to obtain new information [6]. Figure 1 shows three example solutions to the problem of calculating the area of a parallelogram; some are easier than others. Intelligent agents able to select the representation that highlights appropriate features of the problem provide a more valuable solution to a human.

This improved explanatory power has applications across all fields, but we are particularly interested in education. Intelligent tutoring systems could present students with interpretable solutions, and provide hints to struggling students on how to progress by writing the problem in a different way. Similarly, interactive theorem provers could present new perspectives on a problem through re-representation.

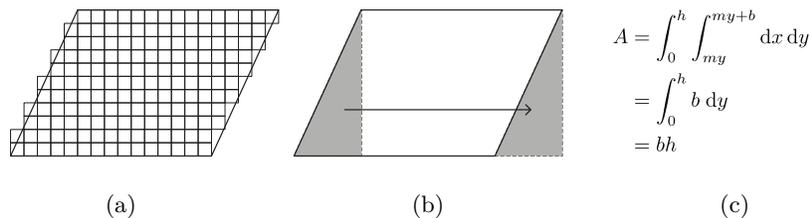


Fig. 1. Three alternative proofs for the area of a parallelogram: (a) depicts approximating the area with small squares; (b) “cuts” a right triangle and translates it to form a rectangle; and (c) uses integration—it is a formalised version of (a).

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In studying what makes a representation appropriate for solving a problem, we aim to address the following questions:

1. How can we identify key properties of representations and problems?
2. Can we exploit these properties to guide representation change towards “easier”—a user-dependent metric—solutions?
3. Can this process be automated to construct a heterogeneous proof assistant?

This paper presents a conceptual overview of our proposed solution, as well as an example analysis of our problem from Figure 1.

Properties Reasoning agents can be equipped with a set of representations, each designed to emphasise specific properties. Similarly, the problem we are trying to solve has properties that may or may not line up with any given representation. Because several representations might be characterised by the same subset of properties, we construct a weighted optimisation function favouring some properties over others. The representations that maximise this function are potential candidates for re-representation; adjusting the weights offers different representation options.

Consider a representation as any (two-dimensional) artefact created to convey meaning and allow inference [2]. This definition allows us to discuss both sentential (natural and formal languages, equations, etc.) and diagrammatic representations. Multiple frameworks exist to classify representations [4], most from a computational perspective [1, 5]: can this representation be used for inference, and is it computationally feasible? Thus, we categorise properties of representations and problems into two classes: formal, about the nature of the content; and cognitive, about the cognitive processes demanded by the representation. Four example properties related to our problem in Figure 1 are:

Symmetry (Formal) Symmetry is a common property; parallelograms have rotational symmetry.

Precision (Formal) The solution to the problems must be precise.

Working memory (Cognitive) Problems are usually easier to solve if they have a low working memory load. Memory load might increase due to poor information indexing, excessive labelling, or many other reasons.

Generality (Cognitive) Representations that are reusable in other problems have implicit benefits for users due to amortisation of comprehension costs.

Example analysis The problem of calculating the area of a parallelogram can be solved in numerous ways, including but not limited to: subdivision into tiny squares, slicing and translating a right-angled triangle, integration, triangulation, treating parallelograms as a special type of trapezium, or “rigorous” natural language. We focus only on the first three, presented in Figure 1.

Squares subdivision Figure 1(a) is low precision, but also has low working memory requirements—despite clutter adding visual noise—and is very general: many other shape areas could be calculated this way. There is no explicit exploitation of symmetry.

Sliding triangle Figure 1(b) is precise, and also has low working memory requirements. It exploits aspects of parallelogram symmetry, but this makes it a specialised technique; areas cannot typically be calculated this way.

Integration Figure 1(c) is precise, and very general, but it has a high working memory load and makes no appeals to the symmetry of the parallelogram.

Each representation is a valid way to solve the problem, but intuition tells us they are not equally good solutions.

Weighting properties Favourable properties of a problem-solving strategy are often generality and “simplicity”; we will use low working memory load as a stand-in for simplicity. Prioritising these properties favours the subdivision method. But this representation suffers from being too imprecise: if we judge the “balance” of over- and under-flow incorrectly while tiling the parallelogram, we end up with the wrong answer.

Alternatively, we could prioritise precision and generality: not only do we want to solve many problems in the same way, we want to be correct when doing so. This would favour selection of the integration technique. The area of any shape definable by equations can be rigorously calculated this way. But the working memory load is very high: there are many symbols, notations, and we introduce constants not mentioned in the problem.

We could drop the generality requirement, focusing instead on a solution that is both precise and has low working memory load. This leads to the sliding triangle representation. It does not apply well to most shapes, but is more precise than subdividing into squares and less memory-intensive than integration.

Conclusions and future work We proposed to investigate properties to guide the choice of representation for a problem. Our small example shows the potential of how systems could adapt to a range of users with a single mechanism, choosing representations to suit both the problem and the problem solver.

Next, we will address questions about which properties to measure, how to measure them, and their relative weightings. We will computationally model our findings in an automated reasoning system in the hope to achieve appropriate suggested representation recommendations that lead to lower time-to-solution processes resulting in outputs that have shorter or clearer proofs, or are solutions for otherwise unsolved conjectures.

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Innovative Use of Diagrams and Inference in Wallis

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1 My topic of dissertation

Twenty century philosophy of mathematics has focused on logical foundations of axiomatic systems and formal arguments. This logic-based approach has considered only deductive proof and axiomatic systems as legitimate objects of philosophical analysis, both elements codified in a symbolic linguistic manner, letting aside other forms of representation as well as non-deductive inferences. The syntactic deductive presentation, constitutive of formal logic, requires to make explicit all relevant steps in a proof guaranteeing formal rigor. This idea crystallizes in the modern axiomatic conception of rigor which assumes that mathematical problems are satisfactorily solved only when the results obtained can be inserted within a deductive system³. By focusing on foundation rather than generation of mathematical knowledge the standard approach cannot provide a characterization of the mathematical research.

More recently, and despite the apparent persistence of this perspective, some critical voices have been insisting upon the value of paying closer attention to the work of the research mathematician focusing on the specificity of his problem-solving activities in the relevant context of work [5]. As Grosholz notes, the fact is that “(t)he outcome of mathematical progress is not always, and perhaps only rarely, an axiomatized system, where solved problems recast as theorems follow deductively from a set of special axioms, logical principles, and definitions” [2, p. 46]. According to the critical line of research developed by Grosholz [1][2] mathematical research cannot be identified exclusively with formal reasoning, and is best characterized by problem-solving activities which make use of a variety of modes of representation. Grosholz shows how the research mathematician uses and develops different systems of representation such as symbolical notations, equations, and diagrams articulated in deductive and non-deductive arguments in mathematical discourse in order to deal with mathematical problems. From this point of view, the appeal to modes of representation other than symbolic notations, as well as ampliative patterns of reasoning, are constitutive of mathematical research. Therefore, it appears as imperative to focus more closely

³ A view already put forward by Hilbert’s requirement that a problem be considered as solved only in case the solution has been deduced from axioms in a finite number of steps.[3, p. 244]

on problem-solving strategies “in the making”, in order to clarify the epistemic role played by these elements neglected by previous approaches.

2 The approach being taken

Our research can be located in the context of what has been called “philosophy of mathematical practice” [5]. Against this background we shall focus on the study of available representations and inferences. The different modes of representation have more or less explicit rules of use that the mathematician learns through his/her work in a given mathematical tradition. Once we conceive a broader view of inference including other kinds of representational objects such as diagrams, sketches, etc. - other than one dimensional symbolic elements arranged deductively -, the rules determining the way in which representations are manipulated can be seen as sanctioning inferential steps in this broader sense. Attending to the fact that problem-solving strategies are designed to deal with mathematical problems of a particular field of inquiry, they should be considered as actually deployed by the research mathematician in his/her context of work. In order to examine the philosophical aspects at the center of our research we shall proceed by focusing on historical case studies.

In our paper we consider a case study selected from John Wallis’s work on quadrature problems [7][6]. We shall focus on Wallis’s exposition in *Arithmetica Infinitorum* (1656), a text that aims to provide an arithmetical method to solve quadrature problems. The interest of the selected case-study resides in two elements that will be the object of this research: on one hand, Wallis’s use of a variety of modes of representation such as algebraic equations, arithmetical series and geometrical diagrams, on the other, the employment that mathematician make of non-deductive inferences. Among the non-deductive inferences, those inferences based-on diagrams deserve special attention due to the fact that their epistemic value seems to lie in a specific kind of spatial configuration. In the case of Wallis, the appeal to a variety of modes of representation and the employment of non-deductive inferences are combined in an innovative method that allows to solve quadrature problems in a general way.

3 The work that has been completed

At an early stage, our investigation was devoted to the analysis of the argumentative structure of Wallis’s method. As a result, we have identified three distinguishable phases. First, the geometrical phase in which Wallis considers the geometrical problem of determination of proportions between the area of two given curves. The second phase, where the problem to be solved is the determination of the ratio between two arithmetical series. In the third phase the results of the two previous stages are related by establishing a correspondence between geometrical and arithmetical items. In a further stage, we have aimed to identify the different modes of representation and inference patterns involved in each of the previous three phases. At the moment, we are focused on considering

the evaluation of the epistemic role of diagrammatic representations. To make our point we are specifically focusing on Wallis's work on the quadrature of conic sections. The main conclusion reached at this stage is that Wallis's novel formulation of quadrature problems is made possible by the method of indivisibles, while his resolution depends on the cartesian approach. Both elements collapse in his conception of conics as planar figures, a view that can only be fully articulated by considering the diagram accompanying the text. Hence, Wallis's method depends on the use of different representations among which diagrammatic representations play an essential role. This partial conclusion suggests the need of going deeper in the study of inferences based on diagrams.

4 The expected contribution of the research

The present study is focused on problem-solving "in the making" and is expected to provide a faithful characterization of how mathematical knowledge grows in the case under consideration. In recent years numerous studies have been made in this direction. However, some scholars have pointed out a number of limitations of current practice-based approaches to provide a strong model of mathematical research [4]. By focusing on the study of available representations and inferences, our investigation intends to provide a tentative frame of analysis which can be extended to other cases.

5 Aspect on which advice is desired

As noted above, given the partial conclusions of our work in progress on Wallis's method and his reliance upon innovative use of different modes of representation among which diagrams play an essential role, it should be of value for our ongoing research to acquire deeper understanding of the conceptual aspects related to the study of context-sensitive inferences as mixed modes of reasoning based on diagrams.

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