Cultivating Diagram Drawing Skills for Math Word Problem Solving Among 1st-Grade Elementary School Students: Making the Link Between Concrete and Abstract Representations

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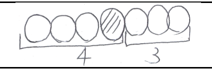
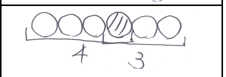
Abstract. Using diagrams is a very efficacious strategy for students in problem solving. However, students tend to experience difficulty in using it as it demands abstraction. Examining how to cultivate a solid foundation for diagram use from an early stage in real educational contexts is important. Thus, in this study, a new 45-minute instruction for 1st-grade elementary school students was collaboratively designed between teachers and a researcher. The way to solve math word problems with diagrams was taught, and making connections between semi-concrete representations (arithmetic learning blocks) and abstract representations (diagrams with circles to represent people/items) was facilitated through explanations and collaborative exercises (peer interaction). The instruction was provided in two classes, which were treated basically the same except in one group concrete objects (blocks) were actively used before drawing diagrams during collaborative problem solving. The students' performance was compared to that of a class without such instruction. The results demonstrated that both spontaneous diagram use and performance in the difficult type problems taught during the instruction were higher in the classes that received the instruction. The class performance in the extension problem was highest in the class that received instruction with the use of concrete objects in collaborative learning situations. The results suggest the importance of providing skills instruction to young learners, and of providing them with the experience of solving problems collaboratively with concrete and abstract representations, which lets students make the link between abstract and concrete representations deeply and internalize the skills more.

1. Introduction and Design Principles of the Instruction

Although the effectiveness of diagram use in problem solving has been demonstrated in many fields (e.g., [1, 2]), students tend to experience difficulty in using diagrams because it demands a kind of abstraction from concrete situations. Lack of skills in constructing abstract diagrams from concrete situations was discussed as a factor that affects spontaneous diagram use [3]. If students have a strong basis for understanding abstract diagrams, diagrams may contribute to enhancing their performance. Therefore, examining how to cultivate skills for understanding and constructing diagrams from an early stage in school education is important. However, many studies about diagram use have been conducted with junior high school students and above (rarely at the early stage of elementary school), and most of them have not been in real classroom contexts. To address those problems, teaching practice was undertaken in one public elementary school in Japan. The author and teachers collaboratively designed a 45-minute instruction. Participants were students in the 1st-grade (n = 79 in total), who were 6-7 years old. The problems used are shown in Figure 1. The students were taught how to use, and the value of using, diagrams in math word problem solving.

Table 1. The Problems and Diagrams Used in Current Study

1 Easy	When lining up, Lisa is the 4th from the front, and there are 3 people after her. How many people in total? (4+3=7)	
2 Easy	When lining up, there are 4 people before Lisa, and she is the 3rd from the front. How many people in total? (4+3=7)	
3 Difficult	When lining up, there are 4 people before Lisa, and 3 people after her. How many people in total? (4+3+1=8)	
4 Difficult	When lining up, Lisa is the 4th from the front, and 3rd from the back. How many people in total? (4+3-1=6)	

The design principles used were: 1) As this was the first time for the students to learn about diagrams representing people with circles (a kind of abstraction was demanded), not only such semi-abstract diagrams with circles, but also concrete representations of small blocks were used to make them deeply understand the meaning of the semi-abstract diagrams. Using the concrete blocks was intended to make them understand better the connection between the situations described in the problems and represented by the diagrams. 2) To make them clearly understand the value of using diagrams, different types of problems like those shown in Figure 1 were used in the one class even though using those problems all in one class is not common in Japan. By solving similar but different types of problems, students might better appreciate the value of diagram use. 3) The class gave students a chance to learn from teachers and to check their comprehension in peer pair work, as well as to solve applied problems collaboratively with diagrams in group contexts.

The flow of the instruction was as follows: Firstly, a teacher used problems 1 and 3 in Figure 1 and modeled on the board how to represent the situation and solve it with small blocks and then diagrams. Students moved their own small blocks on their own desk by following the teacher's movements and drew a diagram together with the teacher. Secondly, students were asked to construct diagrams again by themselves and to explain in pairs to check their comprehension. Thirdly, all types of problems shown in Figure 1 were given to the students and they were asked to solve the problems with diagrams in groups. Students were permitted to use a small magnetic board on which small blocks can stick, and many groups of students put the small blocks collaboratively before they drew the diagrams for each problem. How to solve problems 1–3 was covered in the class, but solving problem 4 was not as time was limited and no group reached that problem. Three days after the class, a post-test was conducted, and each student solved the problems individually (without instruction to use diagrams). This is the flow of "instruction with peer interaction with concrete objects". The flow of "instruction with peer interaction" was basically the same except, before starting to collaborate in solving, students were given 3 minutes to work on their own and draw diagrams by themselves. As a result, most students in this group did not use concrete objects as they already created diagrams on their worksheets. Their collaboration was more sharing of ideas with peers rather than collaboratively constructing diagrams and solving problems. As this study was conducted in a real classroom, using a pure control group was ethically impossible. The class with "no instruction" received instruction on a different math unit and only participated in the post-test for comparison purposes.

2. Results and Discussion

Students' responses to the post-test were analyzed. Spontaneous use of diagrams was counted, and the two classes with instruction were much higher (100%) than the class without instruction (3.6%) was confirmed. Correctness in each problem (see Figure 2) was analyzed and the result showed a significant interaction effect between class and the problem type ($F_{(6,228)} = 6.4$, $p < .001$). The simple main effect analysis revealed that the effect of class was significant only in problems 3 and 4 (difficult type problems, n.s.), and not in problems 1 and 2 (easy type problems, $p < .005$). The multiple comparisons among classes showed that, in problem 3 which was a difficult type and used in the teacher's instruction, the performance was lower only in the class without instruction, but the other two classes did not differ. Also, in problem 4 which was a difficult type and not covered during the class (extension task), the performance was higher in the class "instruction with peer interaction with concrete objects" than the other classes (the effect sizes were .78 and .45, so medium size effects). These results suggest the importance of providing skills instruction about diagram to young learners, and of providing them with the experience of solving problems collaboratively with semi-concrete and abstract representations, which should let students make the link between abstract and semi-concrete representations more deeply and promote internalizing the skills more. This was a practice-based research, so further examination is necessary by using more robust methods to confirm the findings obtained from this study.

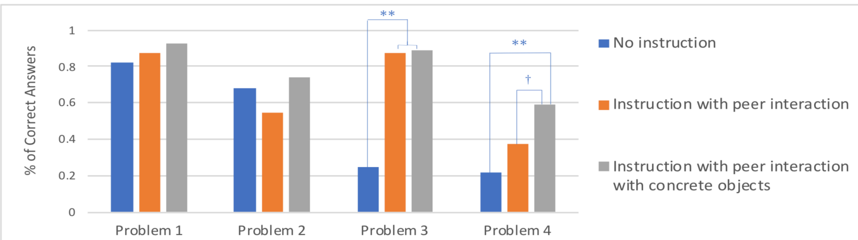


Fig. 2. Percentages of Correct Answer in Each Class at Post-test (**: $p < .01$, †: $p < .1$)

References

1. Hembree, R. (1992). Experiments and relational studies in problem-solving e a metaanalysis. *Journal for Research in Mathematics Education*, 23, 242-273.
2. Ainsworth, S., & Th Loizou, A. (2003). The effects of self-explaining when learning with text or diagrams. *Cognitive Science*, 27, 669-681.
3. Uesaka, Y., & Manalo, E. (2012) Task-related factors that influence the spontaneous use of diagrams in math word problems. *Applied Cognitive Psychology*. 26, 251–260.