An exploration into how mathematicians define visual representation terms.

Siân Goodwin

Loughborough University, UK

Abstract. This paper outlines a study into how mathematicians define words like 'diagram' and other visual representation terms. Through an online survey and interview study, we found that these definitions are not only indistinct between terms, but that there are inconsistencies in how mathematicians use each term. From the study, we extract classes of properties and functions associated with the various visual representation terms.

Keywords: Diagram · terminology · definitions · visual representation

1 Introduction

This PhD thesis will explore how different Communities of Practices (CoPs) in mathematics define visual-representation terms. The hope is that this research will provide us with insight into how these CoPs think about these words.

Oftentimes, students have misguided ideas about topics in mathematics because of how mathematics terminologies are used in daily life outside of mathematics – a term that Pimm [2] coined as semantic contamination. That means that by the time students get to university they have to un-learn preconceptions.

The thesis will present a collection of participant-centred, mixed-methods studies. [1]

2 Work completed to date

So far, an online survey has been completed by mathematicians. A selection of 26 objects was presented to the participants, each with a set of options, where the participants were instructed to select all that apply. Figure 1 shows an example of one of these questions.

Also included was an open question that asked each participant to define each of the terms used in the previous questions. This allowed for us to make connections between participants' definitions more generally, and was placed at the end of the survey so that their definition did not skew their choices.

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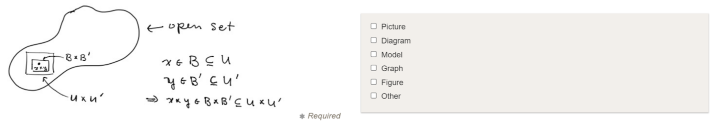

Fig. 1. An example of a tick-box question in the survey.

Additionally, interviews took place with 11 of the participants, which allowed for responses to more complex and deep questions [1]. Participants who were interviewed were presented with their own responses and asked to give justification.

This survey is, at the time of writing, open for university mathematics undergraduates (all years), so that we can compare how mathematicians and their students use the same terms, and work towards a more coherent definition of a diagram so that usage of such terms is more consistent.

Seventy-five mathematicians took part in the survey. To identify the meaning of diagram, two outcomes of the survey were focused on: variables called Endorsement, as a proportion of diagram endorsements across all participants, and Purity, as a proportion of *only* diagram selections to diagram selections overall for each object. These variables were plotted on the same axes against the list of objects; Figure 2 shows the plot for 'Diagram'.

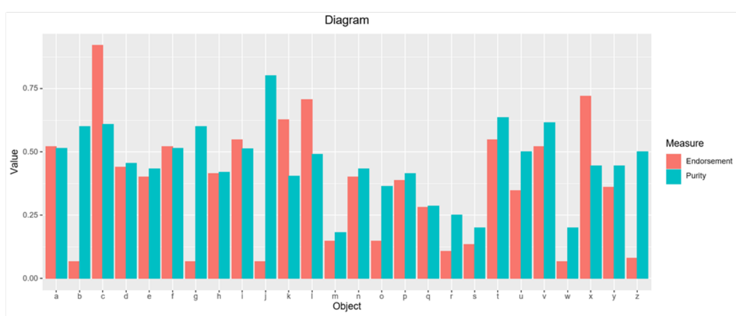

Fig. 2. Plot of Endorsement and Purity for 'diagram' across all objects

The open questions were coded descriptively [3] in that participant responses were summarised while trying to not alter or diminish the participants' intended meaning. The codes were assigned as either a 'Function' or a 'Property'. An example of the Properties and Functions of a diagram are demonstrated in Table 1.

This table lists of all properties and functions selected by at least one of the 75 participants in this study. This means that some of these items in the table are more common than others. Therefore, we reflect on Czocher and Weber's [4] work in suggesting proof as a cluster category, which they take to mean a

Table 1. List of Functions and Properties of diagrams

Function of Diagram	**Property of Diagram**
Accompanies text	
Represents a concept	Symbolic representation
Demonstrates relationships	Accurate/precise
Represents something technical	Labelled
Represents abstract concept	Logical structure
Conveys mathematical system	Simplified representation
Explanation	Has two axes
	Hand-drawn
	Technical illustration
	Formal
	Non-realistic
	Geometric construction

collection of properties (and in our case, functions) that an object *can* satisfy in order to belong to a category. In a similar way, due to no strict consensus reached on how diagram should be defined, we suggest that in future we consider diagram as a cluster category, where we may specify features that are neither necessary nor sufficient in calling an object a diagram.

3 Expected contribution of this research

We propose that this research contributes to several areas of interest. Firstly, within education research, constructing definitions of terms like diagram means that teachers and researchers can be more precise in their usage of those terms, leading to less preconceptions and misconceptions. And while the use of terms like 'diagram' might not be vital to learning mathematics, learning that mathematics requires clear and precise language is important in students' development.

This research also helps us understand how mathematicians use and describe diagrams, which then lends itself to questions about the validity of diagrams in proof. For example, does the way we think about the word 'diagram' influence our decision on what can and cannot be a rigorous proof?

References

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